General

Physics # 108136

INTRODUCTION

Physics and Measurement

Prepared By

Prof. Rashad Badran

Dimensional Analysis

❑ Formula can be checked by dimensional analysis:

Dimensions of physical quantities must be consistent in an equation,

(*i.e.* the same on both sides of the equation)

$$\text{Distance } (d) = \text{velocity}(v) \times \text{time}(t)$$

L.H.S of this equation: dimension of distance or [distance]=?

R.H.S of this equation: dimension of vt or $[vt]$=?

Here, if L.H.S=R.H.S, then the equation is dimensionally correct.

Dimensional Analysis

- Dimensions must be consistent

(i.e. the same on both sides of the equation)

$$d = v\, t$$

Dimensions of t is Time: T

Dimensions of d is Length L

Dimensions of v is $[v]$=L/T

Dimensional Analysis

- Dimensions must be consistent

(i.e. the same on both sides of the equation)

$$[d] = [v\ t]$$

$$=[v][t]$$

$$L = \frac{L}{T} \quad T$$

By Prof. Rashad Badran

Dimensional Analysis

- Dimensions must be consistent

(i.e. the same on both sides of the equation)

$$[d] = [v\ t]$$

$$= [v][t]$$

$$L = \frac{L}{T}\ T \quad ✓$$

The equation is dimensionally correct because the dimensions of all terms on both sides of the equation are the same

Dimensional Analysis

Example

Show whether the following equations are dimensionally correct or not:

(a) $v_f = v_i + a_x t$

and,

(b) $v_f^2 = v_i^2 + 2a_x(x_f - x_i)$

where v_f , v_i , a_x , $x_f - x_i$ and t are final velocity, initial velocity, acceleration, displacement and time, respectively.

Dimensional Analysis

Solution:

(a) We need to check the dimension of each term on right-hand-side (R.H.S.) and left-hand-side (L.H.S) of the equation

$$v_f = v_i + a_x t$$

L.H.S: $[v_f] = L/T$

R.H.S: Two terms $\Rightarrow$ $[v_i] = L/T$, AND, $[a_x\ t] = (L/T^2)(T) = L/T$

Thus, L.H.S = R.H.S, and the equation is dimensionally correct

Dimensional Analysis

Solution:

(b) Again we need to check the dimension of each term on right-hand-side (R.H.S.) and left-hand-side (L.H.S) of the equation

$$v_f^2 = v_i^2 + 2a_x(x_f - x_i)$$

L.H.S: $[v^2_f] = (L/T)^2$

R.H.S: Two terms $\Rightarrow [v^2_i] = (L/T)^2$, AND, $[a_x\,(x_f - x_i)] = (L/T^2)(L) = (L/T)^2$

Thus, L.H.S = R.H.S, the equation is dimensionally correct

Dimensional Analysis

- Unknowns in a formula or equation can be found by dimensional analysis:

(Here, at the first beginning L.H.S must be set equal to R.H.S)

Dimensional Analysis

Example

Find the exponents n and m in the formula $x \;\alpha\; a^n t^m$,

where x is in meters a is in m/s^2 and t is in seconds.

Dimensional Analysis

Solution:

The given formula $x \;\alpha\; a^n t^m$ is only correct when R.H.S. = L.H.S.

On L.H.S. : $[x]=L$

On R.H.S.$[a^n t^m]= (L/T^2)^n (T)^m$

Since L.H.S = R.H.S. $\Rightarrow L= (L^n/T^{2n})(T)^m \quad \Rightarrow L=(L^n)(T^{-2n+m})$

$\Rightarrow$ Equate the exponents of L on both sides of equation to get $n=1$.

Also equate the exponents of T on both sides of equation to get the relation $0 = -2n+m$. Thus $m=2$ and our relation is $x \;\alpha\; a\, t^2$

Note: This result represents the well-known equation of motion for a particle moving with uniform acceleration if it starts from rest, namely, $x = at^2/2$. Here, (1/2) is the constant of proportionality which is dimensionless.

Dimensional Analysis

Exercise

Find the exponents n and m in the formula $a \;\; \alpha \; r^n v^m$,

where a is the acceleration of a particle moving in a circle of radius r with a constant speed v.

Answer: $m = 2, n = -1$

Dimensional Analysis

- Dimension or units of an unknown physical constant can be found by dimensional analysis.

(L.H.S must be set at the beginning equal to R.H.S)

Dimensional Analysis

Example

The gravitational force between two objects of masses M and m separated by a distance r is known as the inverse square law and is expressed by: $F = \frac{GMm}{r^2}$, where G is the proportionality constant. If the force F has the SI units of $Kg.\ m/s^2$, what are the units of G.

Dimensional Analysis

Solution:

Since L.H.S = R.H.S of the equation $F = \frac{GMm}{r^2}$

$\Rightarrow$ (Kg. m/s^2)= $[G]$ (Kg)(Kg)/(m^2)

$\Rightarrow [G] = m^3/Kg.s^2$

Or $\Rightarrow [G]=L^3/MT^2$ in terms of the standards L, M and T

Note: G is called the Universal Constant of Gravity.

Dimensional Analysis

Exercise

Find the dimensions of the constants a, b and c in the equation

$x = a + bt^2 + ct^3$, where x is the position of a particle (in meters) and t is the time (in seconds).

Answer: $[a] = m$, $[b] = m/s^2$, $[c] = m/s^3$

Dimensional Analysis: Review

Objective Question

1. Does the dimensional analysis provide the numerical values of constant that may exist in an algebraic expression?

Answer:

No it does not.

Dimensional Analysis

Objective Question

2. Does the dimensional analysis provide the units of any constant that may exist in any physical equation?

Answer:

Yes it does provide the units of any constant in any physical equation .

Dimensional Analysis

Objective Question

3. If an equation is dimensionally correct, does that mean the equation is algebraically correct?

Answer:

No it does not mean that the equation is algebraically correct.

Dimensional Analysis

Objective Question

4. If an equation is not dimensionally correct, does that mean the equation cannot be true?

Answer:

Yes it means that the equation cannot be true.

Dimensional Analysis

Objective Question

5. If the exponents of any physical variables can be obtained from an equation, does that mean the equation must be dimensionally correct?

Answer:

Yes, this means that the equation must be dimensionally correct.

Unit Conversions: Problem-Solving Strategy

Section 1.4

$$5\ \text{kg} = ?\ \text{g}$$

1- Start with the conversion factor: 1000 g = 1 kg

2- Form a unity out of it with the known unit in the denominator:

1 = 1000 g / 1 kg

3- Multiply the units by one:

$$5\ \cancel{\text{kg}} \ast (1000\ \text{g} / 1\ \cancel{\text{kg}}) = 5000\ \text{g}$$

Unit Conversion

Example

The official world land speed record is 1228.0 km/h, set on October 15, 1997, by Andy Green in the jet engine car Thrust SSC. Express this speed in meters per second.

Step 1: 1 $km = 1000\ m$, 1 hour = 3600 second

Step 2: 1 = 1000m/km, 1 = 1 hour/3600 sec

Step 3: Multiply

1228 km/h * 1 * 1

= 1228 km/h *1000$m/1km$*1h/3600s

= 1228 * 1000m * 1/3600s

= 341.11 m/s

Uncertainty (Errors)

What is the width of this room?

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10 *meters*

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What is the width of this room?

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But it can be 10.2 *meters* or 9.8 *meters*.

Uncertainty (Errors)

What is the width of this room?

10 $meters$

But it can be 10.2 $meters$ or 9.8 $meters$.

We say:

width = 10 $meters$ with an error of ± 0.2 $meters$

± 0.2 is called the error or the uncertainty.

Uncertainty (Errors)

What is the width of this room?

w = 10 *meters* $\qquad\qquad$ Δw = 0.2 *meters*.

Absolute error: Δw = 0.2 *meters*

Uncertainty (Errors)

What is the width of this room?

$w = 10 \textit{ meters}$ $\Delta w = 0.2 \textit{ meters}$.

Absolute error: $\Delta w = 0.2 \textit{ meters}$

Fractional error: $\Delta w/w = 0.2/10 = 0.02$ (no units)

Uncertainty (Errors)

What is the width of this class room?

w = 10 *meters*   Δw = 0.2 *meters.*

Absolute error: Δw = 0.2 *meters*

Fractional error: Δw/w = 0.2/10 =0.02 (no units)

Percentage error: (Δw/w)*100%= 2% (no units)

Significant Figures

Using a ruler, measure the length of the book.

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$$L = 25.3\ cm$$

There is no doubt about the first digit (2)

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Using a ruler, measure the length of the book.

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There is no doubt about the second digit (5)

The third digit can be different but not totally wrong

Significant Figures

Using a ruler, measure the length of the book.

$$L = 25.3\ cm$$

There is no doubt about the first digit (2)

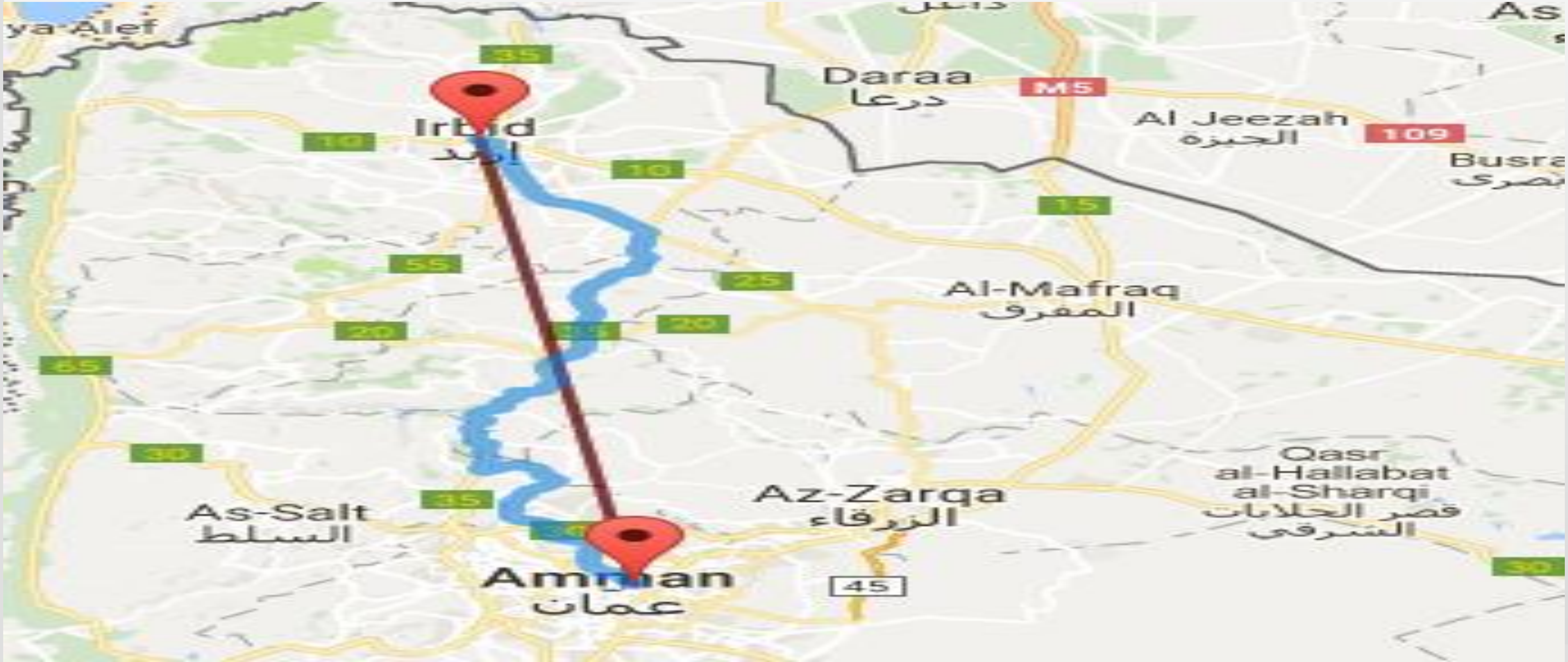
There is no doubt about the second digit (5)

The third digit can be different but not totally wrong

➔ three significant digits

Significant Figures

What is the distance between Amman and Irbid?



Significant Figures

What is the distance between Amman and Irbid?

$92.21 \ km$

Significant Figures

What is the distance between Amman and Irbid?

Significant

$$92.21 \ km$$

Significant

Significant Figures

What is the distance between Amman and Irbid?

Significant

92.21 *km*

Significant

There is some error in this figure but within the value ➔ significant

Significant Figures

What is the distance between Amman and Irbid?

Significant

$$92.21 \ km$$

The error in the measuring process is much bigger than this digit ➔ not significant

Significant

There is some error in this figure but within the value ➔ significant

Significant Figures

What is the distance between Amman and Irbid?

➔There are three significant figures in this number.

$$92.2\ km$$

Notes: (1) The number of significant figures in the final answer of the product or division of different quantities is the same as that in the quantity having the smallest number of significant figures.

(2) The number of decimal places in the final answer of addition or subtraction is equal to the smallest number of decimal places of any term in the process of addition or subtraction